Life of Fred®

Trigonometry

Expanded Edition

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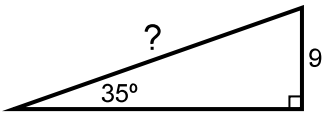
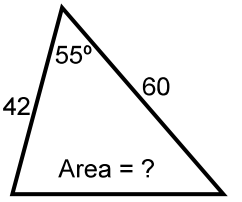
Polka Dot Publishing

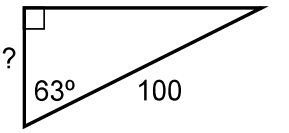
What is Trig All About?

Trigonometry plays with triangles. Mostly right triangles. *Trigon* means triangle and *metry* means measuring (in Greek). Someone probably stuck the "o" in *trigon*-o-*metry* to make it easier to pronounce.

By the end of the first chapter of this book you'll be able to find the quantities indicated by a question mark:







and you'll know the first of the three major trig functions (the sine function). That's the first nine pages of the book.

The rest of the book pretty much flows naturally from those first nine pages. If you looked at the definition of the sine function in Chapter 1 for several minutes, you could predict how the cosine and tangent functions would be defined in Chapter 2. The only thing you wouldn't know is their names.

In Chapter 3 through Chapter 9, we take the concepts of sine, cosine, and tangent and stretch them like taffy.

In Chapter 1 we were taking the sine of the acute angles in a right triangle. In Chapter 3 we wonder what the sine of 110º would equal.

In Chapter 4 we find the basic algebraic facts about sine, cosine, and tangent, such as $(\text{sine of A})^2 + (\text{cosine of A})^2 = 1$ for every angle A. These basic facts will be used later in calculus.

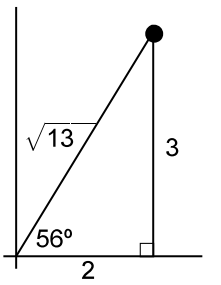
In Chapter 5 we invent a new way to measure angles. Instead of talking about 30º, we have $\pi/6$ radians.

Chapter 6: We put sines, cosines, and tangents in algebra equations and solve them.

Chapter 7: The trig functions are used in triangles that are not right triangles.

Chapter 8: We turn the sine, cosine, and tangent functions inside out by finding their inverses. Back in algebra we knew that if h(Meddie) = apple pie, then the inverse function, h^{-1}, would give us h^{-1}(apple pie) = Meddie.

Chapter 9: We locate the point (2, 3) on a graph using angles and lengths instead of just lengths. Instead of saying that the point is two units to the right and three units upward, we'll say that it is roughly 3.6 units from the origin at an angle of approximately 56º.



The real surprise comes in the last chapter.

All the Chapters from 2 through 9 you might have been able to predict, but not Chapter 10.

In that last chapter we stir together parts of what we've learned in trigonometry so far and come up with the answer to $\sqrt[5]{1}$. Not just the answer $\sqrt[5]{1} = 1$ that you know from algebra. We arrive at five *different* answers. Probably less than 2% of all college graduates can name those five numbers.

In the last question of the last *Your Turn to Play*, you will find the million different answers to $\sqrt[1,000,000]{i}$ and plot all your answers.

i is equal to $\sqrt{-1}$

(It will be easy to do.)

A Note to Students

It is Tuesday evening. Fred is coming back to KITTENS University after a two-day bus trip. Fred celebrated his sixth birthday last Friday. Going from Tuesday evening to Wednesday night in his life, you will learn all of trigonometry.

Reading the adventures in the life of Fred can be done at 1,000 pages per hour or whatever your normal reading speed is. However, the rate at which most people read and understand new material in mathematics is a bit slower than a thousand pages per hour. You are doing well if you learn the trigonometry at about two pages per hour. Using a calculator, we find that works out to about thirty minutes per page.

One of the nice things about mathematics is that there aren't that many pages. I saw in my college bookstore years ago the required reading list for one English course. I think the course dealt with eighteenth-century British novels. There were about a dozen novels the students had to buy—just for one course. My eyes would start to squeak reading that many pages.

Throughout this book are sections called *Your Turn to Play*, which are opportunities for you to interact with the material. Complete solutions are given for all the problems in the *Your Turn to Play* sections, but just reading the problems and the solutions without working them out for yourself really won't work (unless you have an IQ above 150).

If you would like to learn trigonometry, the general rule is easy: Personally work out each of the problems before you look at the solutions I supply.

After 10 chapters you will have mastered all of trig.

Just before the Index is the **A.R.T.** section. This section very briefly summarizes every part of trigonometry. If you have to review for a final exam or if you want to quickly look up some topic eleven years after you've read this book, the **A.R.T.** section is the place to go.

A.R.T. = **A**ll **R**eorganized **T**ogether.

1. The **A.R.T.** section I just mentioned.
2. Motivation. When I taught math in high school and in college, the question that I and every other math teacher received was, "When are we ever gonna use this stuff?"

The *Life of Fred* series is a direct response to that question.

- ✓ Every piece of math first happens in Fred's life.
- ✓ He needs the math.
- ✓ Only then do we present the math.

This is true from the earliest books in the series, in which we first encounter 3 + 4, up through all of calculus. In Chapter 23 in calculus, Betty is driving Fred to their favorite pizza place (PieOne Pizza). There is a heavy wind and Betty tries to figure out how much work her car must do to push through that wind. She does a huge computation.* Fred uses Green's theorem and reduces the whole thing down to

$$\int_{(0,0)}^{(\pi,1)} (\cos x + 6y)dx + (6x + e^y)dy = \phi\ \pi \qquad -\varphi(0,0) = 6\pi + e - 1$$

Fred's One-Liner

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* I don't want to frighten you, so don't look at this very long.

$\int_C (\cos x + 6y)\,dx + (6x + e^y)dy$ where C is the three line segments (0, 0) to (5, 0)
(5, 0) to (5, 1) and
(5, 1) to (π, 1)

which yields three line integrals:

where the parameters of the first integral, for (0, 0) to (5, 0), are x = t and y = 0 and t goes from 0 to 5;
where the parameters of the second integral, for (5, 0) to (5, 1) are x = 5 and y = t and t goes from 0 to 1;
where the parameters of the third integral, for (5, 1) to (5, π) are x = t and y = 1 and t goes from 5 to π,

(and if we call the three line segments C_1, C_2, and C_3)

$$= \int_{C_1} (\cos x + 6y)\,dx + (6x + e^y)dy + \int_{C_2} (\cos x + 6y)\,dx + (6x + e^y)dy + \int_{C_3} (\cos x + 6y)\,dx + (6x + e^y)dy$$

and now we compute each of these integrals.

Betty's Napkin Computation

I know that that calculus stuff looks scary now, but when you are in fourth semester calculus, Fred's One-Liner solution will be a r-e-l-i-e-f.

3. Complete. *Life of Fred: Trig* has all of trigonometry—more material than is normally presented in a university classroom.

4. More than complete. There are six optional chapters (1½, 2½, 3½, 4½, 7½, and 9½) that review much of the material from previous math courses. These Looking Back Chapters are placed just before the trig chapters in which the material will be needed. Many readers appreciate the chance to quickly refresh their memories.

5. Much more than complete. We do much more than just math problems in this book.

There is a natural way to learn—and an unnatural way. Sticking a large group of kids in a sit-up-straight-and-be-quiet classroom, giving them a dose of English for an hour, then herding them to a math classroom for a dose of math, is unnatural.

English teachers teach English. History teachers teach history. Auto shop teachers teach auto shop. But who teaches the kids?

Children (and adults!) love to learn. Watch a bunch of eight-year-olds during the summer playing in the back yard. They find bugs (biology). They dig holes (civil engineering). They wonder why the sun doesn't burn up (nuclear physics). They make mud pies (culinary arts).

One subject tumbles into another. And it is fun.

Life of Fred: Trig aims toward that ideal.

✯ The topic of continuous and discrete variables takes us into a half-page discussion of one of the plays of Shakespeare (p. 53).

✯ When Fred is waiting to see the nurse, he imagines that she will be "a cheerful heir to the legacy of Florence Nightingale." We outline why she played such a pivotal role in the history of women working outside the home (p. 276).

✯ Healthy living is mentioned. *Exercise*: Fred starts his Wednesday morning with "his morning jog around the campus" with the result that "Everything felt so wonderful. He was happy to be alive. . . ." *Diet*: Fred had spent the six years of his life living off vending machine food and pizza with his friends. He had drunk a lot of Sluice during those years—a soda with a lot of sugar in it. He is 36 inches tall and weighs 37 pounds. In the opening two pages of chapter eight, nurse Florrie introduces Fred to a drink that he's never had before. Fred really liked it and exclaimed, "This is great. It quenches your thirst and doesn't leave a nasty aftertaste." And Florrie added, "And no beer gut either." The drink

is . . . water. *Dental hygiene*: Each night (p. 41 and p. 406) Fred flosses and brushes his teeth.

✯ Learning English is at least as important as learning trig. Fred owns a llama, which he received at his birthday party in chapter six. At the beginning of chapter seven he spots the current issue of a llama magazine. We explain why it is incorrect to say that he was *anxious* to read it. And we spend a half page (p. 285) describing the positive results of reading the great authors.

✯ Have you ever eaten saltimbocca?

6. Super much more than complete. I remember the times I was a student and the teacher would provide a glimpse of what lay ahead. I treasured those moments, but those times were rare. Most math teachers just present theorem-definition-theorem-definition-theorem-definition-theorem-definition-theorem-definition-theorem-definition-theorem-definition-theorem and I could never get the Big Picture. In the last chapter of this book, we are going to look ahead to each of the 24 chapters of *Life of Fred: Calculus*. We are going to look at all two years of calculus and get a grand overview. In addition, we'll outline what high school math you will need for each chapter.

7. Fun. How many math books have you ever read that claim that they are fun? This book is fun—along with being a lot of work.

Mary Poppins, that great child psychologist, knew the truth: a spoonful of sugar doesn't hurt at all. This book has $C_{12}H_{22}O_{11}$.

Fred will be jumping out of a hot-air balloon in pitch darkness. He will be served a dinner (on the second page of Chapter 1) by his limo driver that is stunning. And Fred boxes "the big G" Philistine for twelve rounds.

8. Short. Yes, short.

In 69 lessons you will finish all of trig. (There are 10 optional Looking Back lessons and 15 lessons Looking Forward to calculus.)

If you did 34½ lessons per day, you would be done in two days.*

9. $39. No other complete trig textbook comes even close.

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* Other options include: A) two lessons per day, which would take you through trig in 34½ days and B) one lesson per day (Monday–Saturday), in which case you would finish in less than six weeks.

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A.R.T. = **A**ll **R**eorganized **T**ogether
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A strong wind blew from the south. That seemed like a good sign to Fred since he was heading north on Highway 135 in Kansas. It wouldn't be long before he'd be back at the university where he lived. *Home*—that had such a sweet sound to it.

He thought back over the last several days. On Friday, his sixth birthday, he had been seized by the induction evaders investigators and had endured 27 hours chained inside a military prisoner transport. He spent the weekend as a soldier down in a U.S. Army camp in Texas. By Monday he had an honorable discharge. The army chaplain paid his bus fare so that he could get back home. On the bus he had made friends with George and Cheryl Mittens, their three daughters, and the girls' four friends. The bus was hijacked and driven to Cuba, Kansas. And finally on Tuesday after another bus ride to the South Kansas library, George, who had become a multi-billionaire on this bus trip, ordered a limo for Fred's trip back to the northern part of Kansas.*

The back of the limo was almost as big as Fred's office at KITTENS University (Kansas Institute for Teaching Technology, Engineering, and Natural Sciences). The interior was all leather and gold with a telephone, a television, and a wet bar.

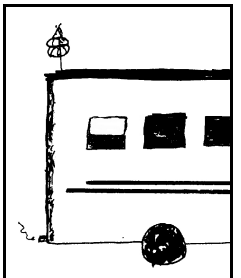
The windows were tinted a dark blue-gray, making it difficult for Fred to see the evening sky.

"Is it okay if I roll down the window to look outside?" Fred asked the driver.

"Of course, Sir," the driver responded. "You may do as you wish. This is your vehicle to enjoy for the trip."

* The adventures of Fred from Thursday afternoon (the day before Fred's sixth birthday) to Monday are told in *Life of Fred: Beginning Algebra Expanded Edition.* Monday and Tuesday are chronicled in *Life of Fred: Advanced Algebra Expanded Edition.*

Fred rolled down the window and was instantly sorry. His lap was filled with snow.



Classic mistake during snowy weather

The bus driver put the limo on AUTOMATIC PILOT and raced back to assist his young passenger. "Don't worry about it, Sir," the driver assured him. "This happens frequently." He vacuumed Fred's lap with a wet/dry vacuum. "Perhaps you would enjoy a bit of dinner before we arrive at KITTENS?"

When Fred looked down at his lap, the driver thought Fred was nodding "yes," and so he began dinner preparations in the limo's kitchen. Fred was hoping that dinner would be a small slice of pineapple pizza. That would hit the spot before he arrived back at his office and could visit the vending machines down the hall. He had recently made it his goal to weigh 40 pounds before he hit puberty. That would mean that he would have to increase his body weight by 10% in the next seven years. The driver/chef placed a large oak table in front of Fred and brought in what he had called "a bit of dinner":

Appetizer

Escargots in an Applewood-smoked Bacon Sauce

Soup

Spring Pea and Squash Blossom Soup with Duck Foie Gras

Salad

Montrachet Goat Cheese Melted over Young Field Greens

Pasta

Smoked Pheasant Ravioli with Fresh Tarragon

Sorbet[*]

Apple-Lime

Entree

Saltimbocca of Salmon in a Bed of Northern Elk Medallions

Dessert

Jell-O®

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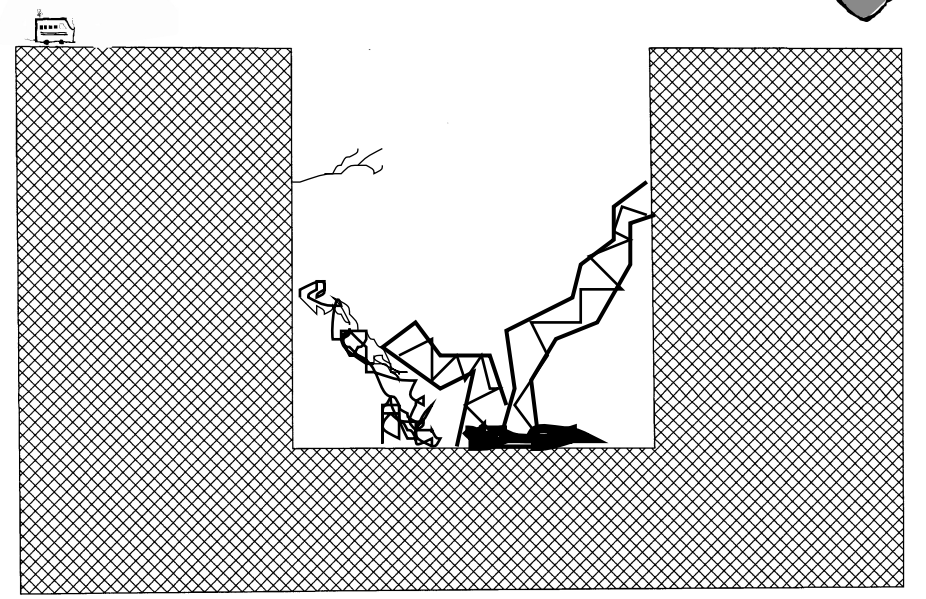
[*] A sorbet is a fruit or vegetable ice which is served before the main course as a palate cleanser. *Webster's Ninth New Collegiate Dictionary* indicates that the correct pronunciation is SOAR-bet. This reflects the fact that sorbet is a word originally from Turkish. *Webster's Tenth* switches to soar-BAY which is the way most people seem to pronounce it today.

Fred sat there stunned. The vending machines down the hall never had anything like this. He carefully nibbled some of the young field greens (avoiding the melted goat cheese).

Suddenly the limo rolled to a stop. The driver looked at his watch and said, "This is too early for the limo to be coming to a stop. We've got another ten minutes before we get to KITTENS." He raced to the front to see why the AUTOMATIC PILOT had stopped the car.

The Troubled Waters Canyon Bridge had completely collapsed. In front of the car was a chasm about a hundred feet across.



Fred and the chauffeur got out of the car and looked at the mess. They could see the lights of the university in the distance.

"Don't worry, Sir," the driver assured Fred. "Acme Ultra Limo Service guarantees that we'll get you to your destination. It's our Gold Service.*"

The driver headed to the trunk of the car and pulled out a large wooden box marked, "Canyon-Fording Emergency #351." Fred watched the driver unpack and inflate a large hot-air balloon.

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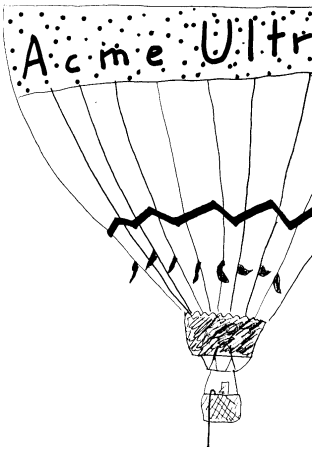
* Acme Ultra Limo is often abbreviated as AU Limo. In chemistry, Au is the symbol for gold.

"If you will just climb in, Sir," the driver said as he lifted his 37-lb. passenger into the balloon's basket.



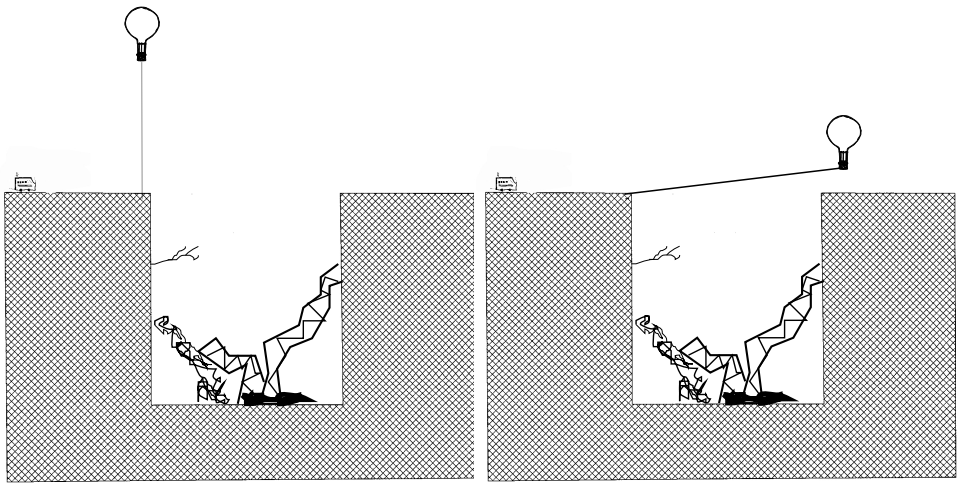
"But, but, but," Fred exclaimed. "I don't know how to fly one of these things!"

"That's quite all right, Sir," the driver answered. "If you would please toss one end of the rope out of the basket and secure the other end, then everything will be quite safe."

Fred did as he was asked. He opened the package marked "Canyon-Fording Emergency Balloon-Tether 120-foot Rope #351A", tossed one end out of the basket and tied the other end around his waist.

The driver looked at what Fred had done and gasped. "Oh no, Sir! I'm afraid I wasn't quite clear. [Some people who serve use the word *quite* quite a lot.] Please affix the rope to the basket. I shall attach the other end to this stake in the ground. Then as the balloon ascends, you shall never be more than 120 feet from me."

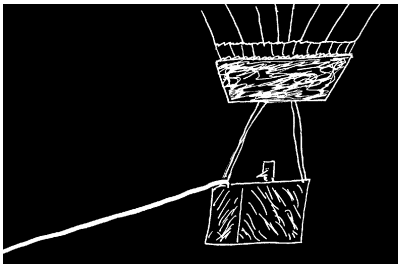
Fred couldn't figure out what was going on. How could going up in a hot-air balloon get him across this chasm?



What Fred thought was going to happen

What really happened

(The driver had read the first seven words of this chapter.) He called out to Fred, "You may jump out of the basket now, Sir."



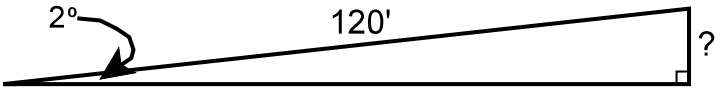
Jump? thought Fred. Where's my parachute? I really can't see how far it is to the ground. It's too dark.

"Driver," Fred called out in the darkness. "How far am I from the ground?"

"I can't tell, Sir," he responded. "It's too dark and you're too far away."

Fred said, "I know the rope is 120 feet long. Can you tell me what my **angle of elevation** is?"

"Very good, Sir." The driver headed to the storage unit in limo and grabbed a protractor,* put it on the ground and measured Fred's angle of elevation (which is how far above the horizontal he was). "Sir," he called to Fred, "Your angle of elevation is two degrees."



Fred needed to know how far the balloon was off the ground (marked by a "?" in the diagram). We have now arrived at the heart of trigonometry. Trig deals with the angles and sides of right triangles.

(Your reading speed should be adjusted appropriately since you're in a more mathematical section. For example, let's read the three sentences of the previous paragraph sssssslllllllllllooooooooowwwwwwwwlllllllllyyyyyyyy and see what we find.

When you read that Fred needed to find the value of the "?," the thought may have come to you as you looked at the diagram that nothing in algebra or in geometry ever showed you how to find the length of the side that is opposite the 2° angle. The sentence, "We have now arrived at the heart of trigonometry," is really a most amazing statement. Five pages into the first chapter and we have a practical example of the use of the first trig function (the sine function) and in Lesson 2 we will have defined it. No other trig textbook that I know of gets to this point this quickly. One trig book takes 165 pages to get to its first application of the sine function. The third sentence, "Trig deals with the angles and sides of right triangles," *defines* trigonometry.

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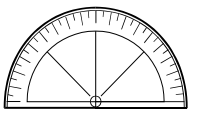
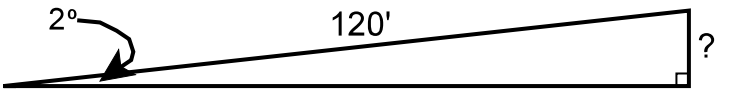
* Protractors are angle-measuring devices. They're usually plastic. You won't need one for trig, but you should have one if you're running a limousine company that offers Gold Service.



Your Turn to Play



1. In the above triangle the angle of elevation is labeled as 2º. When I measure the angle in my drawing, I find it is actually about 6º. Redraw the triangle more accurately. (Please attempt this first on your own before you look at my answer on the next page.)

The following questions are from geometry.

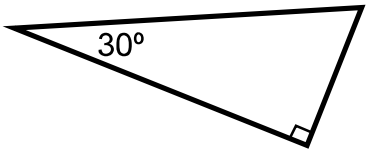
2. What does the little square in the lower-right-hand corner of the above triangle mean?

3. **Acute angles** are angles that are less than 90º. Draw a right triangle that has an acute angle of 45º. (You are not required to own a protractor. Just use your ruler and make a rough drawing.)

4. Make a drawing of a right triangle in which one of the acute angles is approximately 30º.

5. If one acute angle in a right triangle is 30º, what is the measure of the other acute angle?

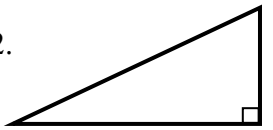
6. A theorem from geometry states, "In any 30–60–90 triangle, the side opposite the 30º angle is half of the length of the hypotenuse." In the following diagram, mark the side opposite the 30º angle with "opp" and the hypotenuse with "hyp".



.....COMPLETE SOLUTIONS.....

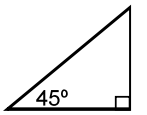
1.

2.



The little square indicates that we are looking at a right angle (= 90°). Triangles which contain a right angle are called right triangles.

3. Your drawing might look like



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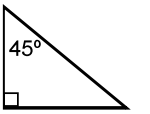

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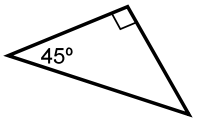
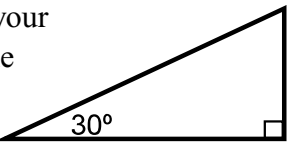


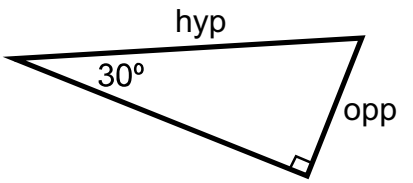
The two shorter sides of the triangle (the legs) you draw should have been roughly equal in length.

4. If you want to see how accurately you drew your right triangle containing a 30° angle, measure the shortest side and measure the hypotenuse (the longest side). According to a theorem from geometry, the shortest side in 30° right triangle is exactly half as long as the hypotenuse.



5. The sum of all three angles in *any* triangle is equal to 180°. If one angle is 90° and another is 30°, that would leave 60° for the third angle. These right triangles in which one of the acute angles is 30° are sometimes called 30–60–90 triangles.

6.



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